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DAMAGE IDENTIFICATION THROUGH FREQUENCY-BASED ANALYSIS FOR STEADY-STATE DYNAMIC RESPONSES

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Abstract. *The presented approach to damage identification is a continuation of research done within the PiezoDiagnostics (PD) project [1]. The general purpose of the PD project was identification of corrosion (or damage of considerable extent) in pipelines. Generation and detection of a global structural mode by piezo-actuators and sensors was tested in the PD project. Perturbations of the mode due to various damage scenarios were investigated. A software tool, based on the Virtual Distortion Method (VDM), was developed [2]. The tool is able to perform damage identification via the solution of an inverse, dynamic problem in time domain thanks to employing gradient-based optimization. A well-calibrated FE model is required for the approach in order to produce meaningful results with experimental data. In this paper, the possibility of carrying out the damage identification in frequency domain will be explored. A dynamic problem with no damping will be considered first. A number of selected excitation frequencies will be the subject of analysis. Steady-state dynamic responses will be provoked and static-like influence matrices in the framework of VDM will be built accordingly. As a consequence, the optimization process in frequency domain is expected to be significantly faster compared to the one analyzed previously in time domain. The newly formulated approach mainly reduce the vast consumption of computational time, observed in the previous approach.*

1 INTRODUCTION

The damage detection systems based on array of piezoelectric transducers sending and receiving strain waves have been intensively discussed by researchers recently. The signal-processing problem is the crucial point in this concept and a neural network based method is one of the most often suggested approaches to develop a numerically efficient solver for this problem.

An alternative approach to the inverse dynamic analysis problem is based on the dynamic VDM (Virtual Distortion Method) concept, making use of a dynamic influence matrix $\mathbf{D}$. Precomputation of the time-dependent matrix $\mathbf{D}$ allows for decomposition of the dynamic structural response into components caused by external excitation in undamaged structure (the linear part) and components describing perturbations caused by the internal defects (the non-linear part). As a consequence, analytical formulas for calculation of these perturbations and the corresponding gradients can be derived. The physical meaning of the virtual distortions used in this paper are externally induced strains (non-compatible in general, e.g. caused by piezoelectric transducers, similarly to the effect of non-homogeneous heating). The compatible strains and self-equilibrated stresses are structural responses to these distortions.

Assuming possible locations of all potential defects in advance, an optimization technique with analytically calculated gradients can be applied to solve the problem of the most probable defect locations. The considered damage can affect the local stiffness as well as the mass distribution modification. It is possible to identify the position as well as intensity of several, simultaneously generated defects.

The proposed methodology can be applied e.g. to corrosion detection (reduction of material thickness), and identification of its location in steel pipelines, using long-distance transmissions of impulses. This time-domain-based methodology of data processing for damage identification (VDM based PD-software, cf. Ref. [2, 4]) fits well to the following technique of measurements (PD-hardware):

i) wave generator produces a low frequency impulse of flexural wave with long-distance propagation,
ii) few well located, distanced sensors collect measurements of frontal section of the transferred wave,
iii) if the received structural response differs significantly from the reference response (for undamaged structure), the collected measurements are transmitted to a computer centre for further data processing (damage identification).

There is a class of problems where a concept similar to the above-mentioned VDM approach, but based on frequency-domain rather than time-domain response can be applied. This numerically economical method can be addressed to problems, where steady-state response can be the basis of dynamic analysis. For example, the following tasks can be solved on the basis of the VDM-F (*Virtual Distortion Method in Frequency Domain*) method:

– remodelling of vibrating system with harmonic excitation in order to reduce vibrations in a selected area,
– identification of material/structural properties on the basis of monitored structural responses for samples of harmonic excitations,
– detection and identification of damages (via inverse dynamic problem) on the basis of monitored structural responses for samples of harmonic excitations.

The objective of this paper is to investigate the third of the above-mentioned problems.

2 PROBLEM FORMULATION

In order to present basic formulas of the VDM-F method, let us focus on quick remodelling of vibrating truss structures under harmonic excitation. Having an existing structure and its parameters we could introduce some modifications to those parameters and then calculate its response, i.e. displacements and internal forces of the modified structure. The general form of equations of motion for a multi-degree of freedom case is given as follows:

$$\mathbf{M}\ddot{\mathbf{u}}(t) + \mathbf{C}\dot{\mathbf{u}}(t) + \mathbf{K}\mathbf{u}(t) = \mathbf{f}(t), \tag{1}$$

where $\mathbf{M}$, $\mathbf{C}$ and $\mathbf{K}$ are mass, damping and stiffness matrices, respectively and $\mathbf{f(t)}$ is the vector of external forces. Each of the above-mentioned matrices represents a set of parameters, which can be modified in the following way:

$$(\mathbf{M} + \Delta\mathbf{M})\,\ddot{\mathbf{u}}(t) + (\mathbf{C} + \Delta\mathbf{C})\,\dot{\mathbf{u}}(t) + (\mathbf{K} + \Delta\mathbf{K})\,\mathbf{u}(t) = \mathbf{f}(t), \tag{2}$$

where $\Delta\mathbf{M}$, $\Delta\mathbf{C}$, $\Delta\mathbf{K}$) describe modifications of the mass, damping and stiffness matrices, respectively. The modification parameters cause non-linear variations of the mass as well as stiffness matrix, which influence the structural response $\mathbf{u}$. The VDM-F based formulation allows to calculate this response quickly (for modified structure) for given (modified) structural parameters. Knowing the responses for original and modified structure, the damage identification process leads to multiple re-computations of dynamic responses with imposed modifications on original structure. This paper is concentrated on the remodelling problem neglecting the damping component.

3 VIRTUAL DISTORTION METHOD IN FREQUENCY DOMAIN

From now on, it is assumed that the structure is subjected to a harmonic excitation. Substituting Eqn:

$$\mathbf{f}(t) = \mathbf{f}\sin(\omega t), \tag{3}$$

to Eqn (1) and (2), the expected response $\mathbf{u}$ can be written in the following form:

$$\mathbf{u}(t) = \mathbf{u}\sin(\omega t) \tag{4}$$

Modifications of stiffness and mass distribution are modelled by *virtual distortions* denoting initial strains in structural elements and virtual forces in structural nodes, oscillating with the same frequency as external excitation:

$$\boldsymbol{\varepsilon}^{\mathbf{0}}(t) = \boldsymbol{\varepsilon}^{\mathbf{0}}\sin(\omega t), \qquad \mathbf{p}^{\mathbf{0}}(t) = \mathbf{p}^{\mathbf{0}}\sin(\omega t), \tag{5}$$

where the first quantity models stiffness, while the second one the mass redistributions, respectively.

Let us call the *modified structure* — a structure in which changes were introduced to the mass and stiffness matrix and the *modelled structure* — a structure in which changes are modelled by virtual distortions, without changing mass and stiffness matrices. The equations of motion for the modified and modelled structures can be obtained introducing virtual distortion component(s) (cf. [3]) to Eqs. (1) and (2):

$$\hat{M}_{ij}\, \ddot{u}_j(t) + G_{\alpha i} \hat{S}_{\alpha\beta}\, G_{\beta j}\, u_j(t) = f_i(t), \tag{6}$$

$$M_{ij}\, \ddot{u}_j(t) + G_{\alpha i} S_{\alpha\beta} \left[G_{\beta j}\, u_j(t) - L_{\underline{\beta}} \varepsilon^0_{\underline{\beta}}(t) \right] = f_i(t) + p^0_i(t), \tag{7}$$

where, $\hat{S}_{\alpha\beta}$ and $S_{\alpha\beta}$ are diagonal matrices with $\hat{S}_{\alpha\alpha} = E_{\underline{\alpha}} \hat{A}_{\underline{\alpha}} / l_{\underline{\alpha}}$ and $S_{\alpha\alpha} = E_{\underline{\alpha}} A_{\underline{\alpha}} / l_{\underline{\alpha}}$, respectively ($E_\alpha$ — Young's modulus, A_α — cross section ($\hat{A}_\alpha$ modified) and l_α length of an element α), L_β — is a vector of lengths of structural elements. In the Eqn (7) (and next formulas) there is no summation over underlined indices. The Greek letters run over structural elements and the Latin ones are related to degrees of freedom of a considered structure. The matrix $G_{\alpha i}$ is a transformation matrix, whose elements are related to cosines of angles between elements and directions of degrees of freedom. The Eqn (7) can be rewritten (using vector of strain ε_α) in the following form:

$$M_{ij}\, \ddot{u}_j(t) + G_{\alpha i} S_{\alpha\beta} \left[L_{\underline{\beta}} \left(\varepsilon_{\underline{\beta}}(t) - \varepsilon^0_{\underline{\beta}}(t) \right) \right] = f_i(t) + p^0_i(t), \tag{8}$$

where the vector of strain ε_α is expressed by relation: $L_{\underline{\alpha}} \varepsilon_{\underline{\alpha}}(t) = G_{\alpha i}\, u_i(t)$.

If the harmonic excitation is investigated, the Eqns (6) and (7) (substituting Eqs (3) and (4)) take the following form:

$$-\omega^2 \hat{M}_{ij} u_j + G_{\alpha i} \hat{S}_{\alpha\beta} G_{\beta j} u_j = f_i, \tag{9}$$

$$-\omega^2 M_{ij} u_j + G_{\alpha i} S_{\alpha\beta} \left[G_{\beta j}\, u_j - L_{\underline{\beta}} \varepsilon^0_{\underline{\beta}} \right] = f_i + p^0_i. \tag{10}$$

In the Eqns (9), (10), the time-dependent components have been eliminated. The displacement depends only on the frequency and the amplitude can be decomposed as follows:

$$u_i = u^L_i + D^\varepsilon_{i\alpha} \varepsilon^0_\alpha + D^p_{ik}\, p^0_k, \tag{11}$$

where: $D^\varepsilon_{i\alpha}$ — influence matrix denoting amplitude of displacement u_i generated by unit, harmonic strain distortion with amplitude $\varepsilon^0_\alpha = 1$ of frequency ω applied in element α. Matrix D^p_{ij} — influence matrix denoting amplitude of displacement u_i generated by unit, harmonic force with amplitude $p^0_i = 1$ of frequency ω applied in j-th degree of freedom.

It is postulated that response of the structure modelled by virtual distortions has to be identical with the response of the modified structure. Therefore, for each element, which is modified, the compatibility of strains and stresses is required:

$$P_\alpha = E_{\underline{\alpha}} \hat{A}_{\underline{\alpha}}\, \varepsilon_{\underline{\alpha}} = E_{\underline{\alpha}} A_{\underline{\alpha}} \left(\varepsilon_{\underline{\alpha}} - \varepsilon^0_{\underline{\alpha}} \right). \tag{12}$$

Assuming that the cross sections of structural elements are modified, the vector of stiffness modification can be expressed as follows:

$$\mu_\alpha = \frac{\hat{A}_{\underline{\alpha}}}{A_{\underline{\alpha}}} = \frac{\varepsilon_{\underline{\alpha}} - \varepsilon^0_{\underline{\alpha}}}{\varepsilon_{\underline{\alpha}}}. \tag{13}$$

The vector μ_α is the vector of structural modification, which involves modification of the mass as well as stiffness matrix). The updated vector of strain ε^0_α can be untainted through multiplying Eqn (11) by $\frac{1}{L_{\underline{\alpha}}} G_{\underline{\alpha}i}$:

$$\varepsilon_\alpha = \frac{1}{L_{\underline{\alpha}}} G_{\underline{\alpha}i} u_i = \varepsilon^L_\alpha + \frac{1}{L_{\underline{\alpha}}} G_{\underline{\alpha}i} D^\varepsilon_{i\beta}\, \varepsilon^0_\beta + \frac{1}{L_{\underline{\alpha}}} G_{\underline{\alpha}i} D^p_{ik}\, p^0_k. \tag{14}$$

For further notations it is advisable to introduce quantities $B^\varepsilon_{\alpha\beta}$ and $B^p_{\alpha k}$ defined as follows :

$$B^\varepsilon_{\alpha\beta} = \frac{1}{L_{\underline{\alpha}}} G_{\underline{\alpha}i} D^\varepsilon_{i\beta}, \qquad B^p_{\alpha k} = \frac{1}{L_{\underline{\alpha}}} G_{\underline{\alpha}i} D^p_{ik}. \tag{15}$$

Substituting Eqn (14) to Eqn (13), the relation between the vector of stiffness modification μ_α and virtual distortions ε^0_α and p^0_i can be determined:

$$\mu_{\underline{\alpha}} \frac{1}{L_{\underline{\alpha}}} G_{\underline{\alpha}i} \left(u^L_i + D^\varepsilon_{i\beta} \varepsilon^0_\beta + D^p_{ij}\, p^0_j \right) = \frac{1}{L_{\underline{\alpha}}} G_{\underline{\alpha}i} \left(u^L_i + D^\varepsilon_{i\beta} \varepsilon^0_\beta + D^p_{ik}\, p^0_k \right) - \varepsilon^0_\alpha. \tag{16}$$

It is convenient to write the above equation in the following form:

$$\left[(\mu_{\underline{\alpha}} - 1) B^\varepsilon_{\underline{\alpha}\beta} + \delta_{\alpha\beta} \right] \varepsilon^0_\beta + (\mu_{\underline{\alpha}} - 1) B^p_{\underline{\alpha}k}\, p^0_k = (1 - \mu_{\underline{\alpha}})\, \varepsilon^L_{\underline{\alpha}}. \tag{17}$$

Equation (17) contains two kinds of virtual distortions ε^0_α and p^0_k. In order to determine those distortions, let us determine the second relationship from Eqns (9) and (10):

$$p^0_i = \omega^2 (\hat{M}_{ij} - M_{ij})\, u_j = \omega^2 \Delta M_{ij}\, u_j, \tag{18}$$

where u_i is described by Eqn (11). Let us assume (for simplicity) the diagonal mass matrices M_{ij} and $\hat{M}_{ij}$. Their difference can be determined as follows:

$$\begin{aligned} \Delta M_{ij} = \sum_\alpha a^\alpha_{ri} \Delta m^\alpha_{rs} a^\alpha_{sj} = \\ = \frac{1}{2} \sum_\alpha (\mu_{\underline{\alpha}} - 1) \rho_{\underline{\alpha}} A_{\underline{\alpha}} l_{\underline{\alpha}}\, a^\alpha_{ri} \delta_{rs} a^\alpha_{sj} = \frac{1}{2} \sum_\alpha (\mu_{\underline{\alpha}} - 1) \rho_{\underline{\alpha}} A_{\underline{\alpha}} l_{\underline{\alpha}}\, M^{(\alpha)}_{ij}, \end{aligned} \tag{19}$$

where a^α_{ir} determines relation between degrees of freedom of a finite element α (indices r, s) and degrees of freedom of the whole structure and $\frac{1}{2} \rho_{\underline{\alpha}} A_{\underline{\alpha}} l_{\underline{\alpha}}\, M^{(\alpha)}_{ij}$ is the global mass matrix calculated for a structural element α.

Thus, let us substitute the Eqn (19) into the Eqn (18):

$$- \omega^2 \left(\frac{1}{2} \sum_{\alpha} (\mu_{\underline{\alpha}} - 1) \rho_{\underline{\alpha}} A_{\underline{\alpha}} l_{\underline{\alpha}} \, M_{ij}^{(\alpha)} \right) D_{j\beta}^{\varepsilon} \varepsilon_{\beta}^{0} + \\ + \left[-\omega^2 \left(\frac{1}{2} \sum_{\alpha} (\mu_{\underline{\alpha}} - 1) \rho_{\underline{\alpha}} A_{\underline{\alpha}} l_{\underline{\alpha}} \, M_{ij}^{(\alpha)} \right) D_{jk} + \delta_{ik} \right] p_k^0 = \\ = \omega^2 \left(\frac{1}{2} \sum_{\alpha} (\mu_{\underline{\alpha}} - 1) \rho_{\underline{\alpha}} A_{\underline{\alpha}} l_{\underline{\alpha}} \, M_{ij}^{(\alpha)} \right) u_j^L . \quad (20)$$

Finally, the formula for determining virtual distortions ε_{β}^0 and p_k^0, taking into account Eqns (17) and (20), can be written:

$$\mathbf{A}\, \mathbf{d^0} = \mathbf{b^L}, \quad (21)$$

where:

$$\mathbf{A} = \begin{bmatrix} (\mu_{\underline{\alpha}} - 1) B_{\underline{\alpha}\beta}^{\varepsilon} + \delta_{\alpha\beta} & (\mu_{\underline{\alpha}} - 1) B_{\underline{\alpha}k}^{p} \\ -\omega^2 \left(\frac{1}{2} \sum_{\alpha} (\mu_{\underline{\alpha}} - 1) \rho_{\underline{\alpha}} A_{\underline{\alpha}} l_{\underline{\alpha}} \, M_{ij}^{(\alpha)} \right) D_{j\beta}^{\varepsilon} & -\omega^2 \left(\frac{1}{2} \sum_{\alpha} (\mu_{\underline{\alpha}} - 1) \rho_{\underline{\alpha}} A_{\underline{\alpha}} l_{\underline{\alpha}} \, M_{ij}^{(\alpha)} \right) D_{jk}^{p} + \delta_{ik} \end{bmatrix},$$

$$\mathbf{d}^0 = \begin{bmatrix} \varepsilon_{\beta}^0 \\ p_k^0 \end{bmatrix}, \qquad \mathbf{b^L} = \begin{bmatrix} (1 - \mu_{\underline{\alpha}}) \varepsilon_{\underline{\alpha}}^L \\ \omega^2 \left(\frac{1}{2} \sum_{\alpha} (\mu_{\underline{\alpha}} - 1) \rho_{\underline{\alpha}} A_{\underline{\alpha}} l_{\underline{\alpha}} \, M_{ij}^{(\alpha)} \right) u_j^L \end{bmatrix}.$$

The virtual distortions ε_{α}^0 and p_k^0 obtained from the above set of equations model modification of cross section area A_{α} (see Eqn (13)) of structural elements. Using Eqn (11) (or Eqn (14)) the displacements field (strain field) can be quickly calculated.

4 NUMERICAL EXAMPLE

Let us illustrate results of the above-discussed modelling of structural parameters on the example of cross section areas of the truss shown in Fig.(1). The original structure has uniform structural parameters, namely:

- Young's modulus $E = 210\, GPa$,
- cross section area $A = 1E - 5\, m^2$,
- density $7800\, \frac{kg}{m^3}$,
- length and height of a single section $l = l_x = l_y = 1\, m$.

The structure is subjected to load F (see Fig.(1)). The determined virtual distortions ε_{α}^0 and p_i^0 will be valid only for this load and can not be used for other load cases.

Two simple numerical examples were calculated to test the modelling of structural parameters. The first case concerns defect in element no. 13. The size of damage is equal to $\mu_{13} = 0.8$,

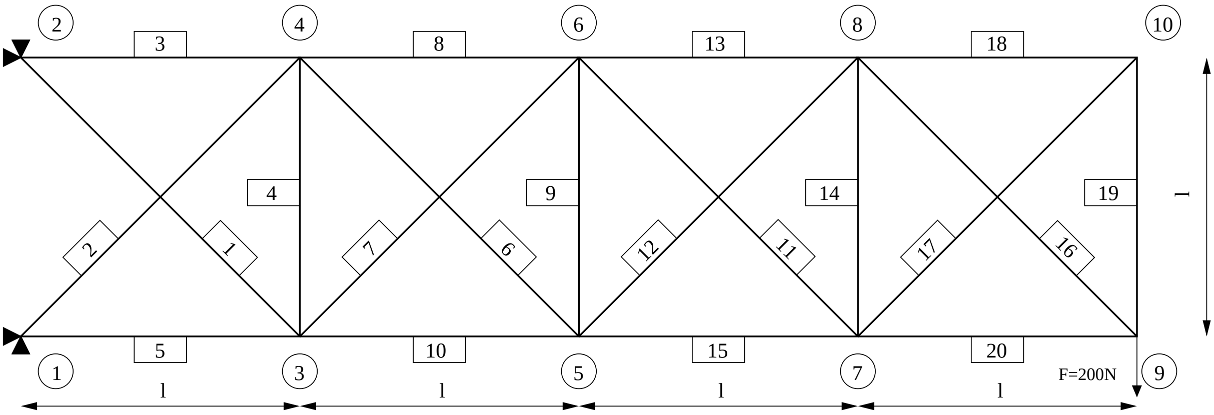

Figure 1. Tested truss structure.

which denotes that the cross section area is reduced to 80% of the initial, undamaged value. In the second case there are different sizes shown in Table (1).

Element no. α	1	8	12	14	18	20
Defect size μ_α	0.20	0.60	0.95	0.75	0.40	0.55

Table 1. Considered defect distribution.

Computations were carried out using VDM-F software (remodelling) and ADINA (modified structure) for those cases. Table (2) contains a comparison of selected nodal displacements calculated for the multi-defect case at selected frequency $\omega = 2\pi f = 100\,Hz$. Table (3) presents similar results for single defect case at various frequencies.

Frequency ω $[Hz]$	Node no.	X-Displacement $[m]$		Y-Displacement $[m]$	
		VDM-F	ADINA	VDM-F	ADINA
100	7	-7.91140E-4	-7.91457E-4	-3.72234E-3	-3.72395E-3
	8	9.86282E-4	9.86689E-4	-3.72868E-3	-3.73030E-3
	9	-8.96878E-4	-8.97221E-4	-5.77493E-3	-5.77734E-3
	10	1.09703E-3	1.09747E-3	-5.73419E-3	-5.73660E-3

Table 2. Comparison of the results: VDM-F vs. ADINA for multi defect case.

Frequency ω $[Hz]$	Node no.	X-Displacement $[m]$		Y-Displacement $[m]$		Non-zero virtual distortions
		VDM-F	ADINA	VDM-F	ADINA	
0	7 8 9 10	-7.17165E-4 7.46327E-4 -7.69387E-4 7.89343E-4	-7.17165E-4 7.46327E-4 -7.69387E-4 7.89343E-4	-2.94144E-3 -2.94922E-3 -4.61264E-3 -4.56962E-2	-2.94144E-3 -2.94922E-3 -4.61264E-3 -4.56962E-3	ε^0_{13}=3.49208E-5
200	7 8 9 10	-1.39382E-2 1.44333E-3 -1.47259E-3 1.51345E-3	-1.39392E-3 1.44344E-3 -1.47269E-3 1.51356E-3	-5.96645E-3 -5.97633E-3 -9.10448E-3 -9.06118E-3	-5.96692E-3 -5.97681E-3 -9.10517E-3 -9.06187E-3	ε^0_{13}=6.03276E-5 p^0_{6x}=-0.359409 p^0_{6y}=0.989675 p^0_{8x}=-0.454021 p^0_{8y}=1.881284
400	7 8 9 10	6.12298E-4 -6.21373E-4 6.09211E-4 -6.28591E-4	6.12523E-4 -6.21571E-4 6.09454E-4 -6.28803E-4	3.05659E-3 3.05270E-3 4.25707E-3 4.29985E-3	3.05744E-3 3.05354E-3 4.25840E-3 4.30118E-3	ε^0_{13}=-1.35139E-5 p^0_{6x}=0.679527 p^0_{6y}=-2.150765 p^0_{8x}=0.76102 p^0_{8y}=-3.746571
800	7 8 9 10	2.01823E-6 1.60308E-5 -3.59114E-5 4.45938E-5	2.03769E-6 1.60461E-5 -3.58890E-5 4.46084E-5	5.34501E-4 5.27712E-4 3.72611E-4 4.16256E-4	5.34533E-4 5.27739E-4 3.72650E-4 4.16296E-4	ε^0_{13}=0.290914 p^0_{6x}=-0.359409 p^0_{6y}=-2.316942 p^0_{8x}=–0.093345 p^0_{8y}=-2.615171

Table 3. Comparison of the results: VDM-F vs. ADINA for single defect case ($\mu_{13} = 0.8$).

5 SENSITIVITY ANALYSIS

In order to determine gradients $\frac{\partial \varepsilon^0_\beta}{\partial \mu_\vartheta}$ and $\frac{\partial p^0_i}{\partial \mu_\vartheta}$ let us differentiate Eqns (17) and (20), respectively:

$$\left[(\mu_{\underline{\alpha}} - 1)B^{\varepsilon}_{\underline{\alpha}\beta} + \delta_{\underline{\alpha}\beta}\right] \frac{\partial \varepsilon^0_\beta}{\partial \mu_\vartheta} + (\mu_{\underline{\alpha}} - 1)B^{p}_{\underline{\alpha}k} \frac{\partial p^0_k}{\partial \mu_\vartheta} = -\delta_{\underline{\alpha}\vartheta}\varepsilon_{\underline{\alpha}}, \tag{22}$$

$$\begin{aligned} &-\omega^2 \left(\frac{1}{2} \sum_{\alpha} (\mu_{\underline{\alpha}} - 1)\rho_{\underline{\alpha}} A_{\underline{\alpha}} l_{\underline{\alpha}} \, M^{(\alpha)}_{ij} \right) D^{\varepsilon}_{j\beta} \frac{\partial \varepsilon^0_\beta}{\partial \mu_\vartheta} + \\ &\quad + \left[-\omega^2 \left(\frac{1}{2} \sum_{\alpha} (\mu_{\underline{\alpha}} - 1)\rho_{\underline{\alpha}} A_{\underline{\alpha}} l_{\underline{\alpha}} \, M^{(\alpha)}_{ij} \right) D_{jk} + \delta_{ik} \right] \frac{\partial p^0_k}{\partial \mu_\vartheta} = \frac{1}{2}\omega^2 \rho_{\underline{\vartheta}} A_{\underline{\vartheta}} l_{\underline{\vartheta}} M^{(\vartheta)}_{ij} u_j. \end{aligned} \tag{23}$$

Similarly to the Eqn (21), the set of equations concerning distortion gradients with respect to the vector of modification parameters is given as follows:

$$\mathbf{A}\,\mathbf{g^0} = \mathbf{b}, \tag{24}$$

where the matrix $\mathbf{A}$ is the same as in Eqn (21), moreover:

$$\mathbf{g^0} = \begin{bmatrix} \dfrac{\partial \varepsilon^0_\beta}{\partial \mu_\vartheta} \\ \dfrac{\partial p^0_k}{\partial \mu_\vartheta} \end{bmatrix}, \qquad \mathbf{b} = \begin{bmatrix} \delta_{\underline{\alpha}\vartheta}\left(\varepsilon^L_{\underline{\alpha}} + B^\varepsilon_{\underline{\alpha}\beta}\varepsilon^0_\beta + B^p_{\underline{\alpha}j}p^0_j\right) \\ \frac{1}{2}\omega^2\rho_{\underline{\vartheta}}A_{\underline{\vartheta}}l_{\underline{\vartheta}}M^{(\vartheta)}_{ij}\,\left(u^L_j + D^\varepsilon_{j\beta}\varepsilon^0_\beta + D^p_{jk}p^0_k\right) \end{bmatrix}$$

Having calculated distortion gradients, the gradient-based formulation for damage identification can be applied.

6 DAMAGE IDENTIFICATION

The result of the damage identification indicate the severity of damaged structural element(s) and their localisation. This inverse problem leads to minimization of a suitable objective function. The objective function has to depend on structural modification parameters to be detected. The accuracy of the result is related with the number of measured responses (sensors). Let us propose the objective function as follows:

$$f = (\varepsilon_\psi - \varepsilon^M_\psi)(\varepsilon_\psi - \varepsilon^M_\psi), \tag{25}$$

where $\varepsilon_\psi = \varepsilon_\psi(\mu_\vartheta)$ is the vector of strain in the considered structure with internal (unknown) defects μ_ϑ modelled by virtual distortions (ε^0_α, p^0_i) and ε^M_ψ is the measured response — in this case obtained numerically — of the damaged structure. In the above formula index ψ runs over selected structural elements. To minimize the objective function (25), the steepest descent method can be used. To this end, let us calculate the gradient of the objective function $\frac{\partial f}{\partial \mu_\vartheta}$:

$$\frac{\partial f}{\partial \mu_\vartheta} = \frac{\partial f}{\partial \varepsilon_\psi}\,\frac{\partial \varepsilon_\psi}{\partial \varepsilon^0_\alpha}\,\frac{\partial \varepsilon^0_\alpha}{\partial \mu_\vartheta} + \frac{\partial f}{\partial \varepsilon_\psi}\,\frac{\partial \varepsilon_\psi}{\partial p^0_i}\,\frac{\partial p^0_i}{\partial \mu_\vartheta}. \tag{26}$$

In the Eqn (26) the distortion gradients are determined from Eqn (24). The partial derivatives $\frac{\partial \varepsilon_\psi}{\partial \varepsilon^0_\alpha}$ and $\frac{\partial \varepsilon_\psi}{\partial p^0_i}$ can be obtained through differentiation of Eqn (14), accounting for Eqn (15):

$$\frac{\partial \varepsilon_\psi}{\partial \varepsilon^0_\alpha} = B^\varepsilon_{\psi\alpha}, \qquad \frac{\partial \varepsilon_\psi}{\partial p^0_i} = B^p_{\psi i}, \tag{27}$$

Now, the gradient of the objective function can be expressed as follows:

$$\frac{\partial f}{\partial \mu_\vartheta} = \nabla_\vartheta f = 2\left(\varepsilon_\psi - \varepsilon^M_\psi\right)\left[B^\varepsilon_{\psi\beta}\frac{\partial \varepsilon^0_\beta}{\partial \mu_\vartheta} + B^p_{\psi k}\frac{\partial p^0_k}{\partial \mu_\vartheta}\right]. \tag{28}$$

The vector of modification parameters is determined iteratively according to the formula:

$$\mu_\alpha^{(s+1)} = \mu_\alpha^{(s)} - \Delta f^{(s)} \frac{\nabla_\alpha f^{(s)}}{\left[\nabla_\alpha f^{(s)}\right]^T \nabla_\alpha f^{(s)}}, \tag{29}$$

with *a priori* assumed original vector of modification parameters $\mu_\vartheta^{(0)}$ (eg. for undamaged structure), where s denotes the current iteration, $s+1$ the next one and the step length Δ is appropriately selected from the interval $(0, 1)$.

7 SUMMARY AND CONCLUSIONS

The Virtual Distortion Method in frequency domain (VDM-F) is a useful tool to investigate steady-state problems. Static-like influence matrices are built only once for each frequency. This allows to quick calculation of the updated responses for given modification parameters, modelled by two kinds of virtual distortions. For the re-modelling problem, two simple examples with one and multi-defect case have been demonstrated.

The presented damage identification methodology requires comparison of the reference response(s) (referring to for undamaged structure) and the damaged one. The optimization process in frequency domain based on VDM-F is expected to be significantly faster compared to the one analyzed in time domain.

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